Solution Ex. 1

$\frac{a+b}{2} > \sqrt{ab}$ ... square both sides

$\frac{(a+b)^2}{4} > ab$ .... expand $(a+b)^2$

$\frac{a^2+2ab+b^2}{4} > ab$ ... multiply both sides by 4

$a^2+2ab+b^2 > 4ab$ ... subtract $4ab$ on both sides

$a^2-2ab+b^2 > 0$ ... use formula for square of a difference

$(a-b)^2 > 0$ ... this is an expression which is always true, since squares of positive or negative numbers are always larger than zero

proven!

Formulas and rules needed:

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

If you modify any equation or inequality you have to do perform all operations on both sides. This will keep the equation or inequality valid.

In case of inequalities sometimes care must be taken with respect to negative numbers. If you multiply with negative number > change to < and vice versa.

Solution of Ex. 2

$$g = 5$$

$$f = \frac{1}{\sqrt[12]{2}} = 0.943874$$

Each fret distance is by a factor f smaller than the previous distance. Thus

first is g
second is $g \cdot f$
third is $(g \cdot f) \cdot f = g \cdot f^2$
fourth is $(g \cdot f^2) \cdot f = g \cdot f^3$
....

To get the distance we have to sum up

$$s = g + g \cdot f + gf^2 + gf^3 + \dots + gf^{23}$$

$$= g\frac{f^{24}-1}{f-1} \quad \text{using a formula found in any textbook}$$

$$= 5\frac{\left[\left(\frac{1}{\sqrt[12]{2}}\right)^{24} - 1\right]}{\left[\frac{1}{\sqrt[12]{2}} - 1\right]} = \frac{5 \cdot 0.75}{1 - \frac{1}{\sqrt[12]{2}}} = \underline{\underline{66,8}}$$

It's a good exercise to derive the above formula for an arbitrary number of frets n

$$\# \begin{cases} -s = -g \mp gf \mp gf^2 \mp \dots \mp g \cdot f^{n-1} & \vert \cdot -1 \\ s \cdot f = \quad g \cdot f + gf^2 + \dots + gf^{n-1} + gf^n \end{cases}$$

$$sf - s = -g + gf^n$$

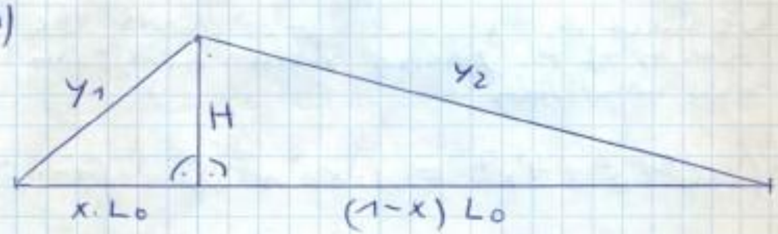
$$s(f-1) = g(f^n - 1)$$

$$s = g\,\frac{f^n - 1}{f - 1}$$

Basic idea is to multiply the sum another time by factor f and subtract the original sum. Then most terms cancel out and you can easily find s by some arithmetic operations!

solution of Ex. 3

a)



$y = y_1 + y_2$ ... total length of stretched string

$y = \sqrt{x^2 L_0^2 + H^2} + \sqrt{(1-x)^2 L_0^2 + H^2}$ ... use Pythagoras... to find $y(x)$

b) Now find for given H and L_0 the minimum for the elongation of the string. To find the minimum of a function we set first derivative to zero. This gives strictly speaking the condition for an extreme value which can be minimum or maximum. For minimum additionally the second derivative must be >0 at the position of the extremum.

$$\frac{dy}{dx} = \frac{1 \cdot \cancel{2} x L_0^2}{\cancel{2}\sqrt{x^2 L_0^2 + H^2}} + \frac{\cancel{2}(1-x)\, L_0^2}{\cancel{2}\sqrt{(1-x)^2 L_0^2 + H^2}} = 0$$

... factor 2 cancels out and we can additionally divide by L_0^2 and subtract the second term on both sides. Thus he cancels out on left hand side and appears on right hand side with negative sign.

$$\frac{x}{\sqrt{x^2 L_0^2 + H^2}} = \frac{-(1-x)}{\sqrt{(1-x)^2 L_0^2 + H^2}}$$

... then we take squares on both sides

$$\frac{x^2}{(x^2 L_0^2 + H^2)} = \frac{(1-x)^2}{((1-x)^2 L_0^2 + H^2)}$$

... then we get rid of nominators by multiplying both sides of the equation with both of them ...

$$x^2\left[(1-x)^2 L_0^2 + H^2\right] = (1-x)^2\left[x^2 L_0^2 + H^2\right]$$

... we multiply x^2 and $(1-x)^2$ with the contents of the squared brackets on left and right hand sides

$$x^2 \cancel{(1-x)^2 L_0^2} + H^2 x^2 = \cancel{(1-x)^2 x^2 L_0^2} + (1-x)^2 H^2$$

... we are lucky: one ugly expression cancels out ...
... to simplify further we can divide by H^2 on both sides ... this yields finally

$$x^2 = (1-x)^2$$
$$x^2 = 1 - 2x + x^2$$
$$0 = 1 - 2x$$
$$2x = 1$$
$$\underline{\underline{x = \frac{1}{2}}}$$

This means that the elongation of the string is smallest, ~~at~~ if we plug or pick the string at $\frac{L_0}{2}$. At this position the likelihood for rupture is clearly also smallest.

solution of Ex. 4

$$f_H = \frac{1}{\sqrt{\frac{a \cdot b}{5}}} = \sqrt{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = f_p$$

$$\frac{5}{ab} = 2\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = 2\left(\frac{1 \cdot b^2}{a^2 b^2}+\frac{1 \cdot a^2}{b^2 a^2}\right)$$

$$\frac{5}{ab} = 2\,\frac{(b^2+a^2)}{a^2 b^2} \qquad \Big| \cdot (a^2 b^2)$$

$$5ab = 2(b^2+a^2)$$

$$2{,}5\,ab = b^2+a^2$$

$$0 = a^2 - 2{,}5\,ab + b^2 \qquad \Big| : b^2$$

$$0 = \left(\frac{a}{b}\right)^2 - 2{,}5\left(\frac{a}{b}\right) + 1 \qquad \text{with } x = \frac{a}{b}$$

$$0 = x^2 - 2{,}5x + 1$$

$$x_{1/2} = \frac{+2{,}5 \pm \sqrt{\frac{25}{4}-4}}{2}$$

$$= \frac{2{,}5 \pm 3}{2} = \begin{cases} \frac{5{,}5}{2} = \underline{\underline{2{,}75}} \\ -\frac{1}{4} \quad \text{(makes no sense)} \end{cases}$$

The formula for solution of quadratic equations

$$A\,x^2 + Bx + C = 0$$

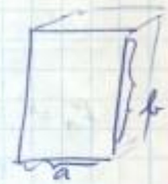
yielding

$$x_{1/2} = \frac{-B \pm \sqrt{B^2 - 4A \cdot C}}{2A}$$

is needed.

Solution of Ex. 5

$S = a \cdot b \quad \rightarrow \quad b = \frac{S}{a}$



$f = A \sqrt{\frac{1}{a^2} + \frac{a^2}{S^2}}$

Since A and S is fixed, frequency f is now a function of a. I.e., we could set $f = y$ and $a = x$ to get a function $y(x)$, but this is just different naming $f(a)$ is completely the same.

To find the maximum we have again to set first derivative to 0.

$$\frac{df}{da} = A \cdot \frac{-2a^{-3} + \frac{2a}{S^2}}{2\sqrt{\frac{1}{a^2} + \frac{a^2}{S^2}}} = 0$$

By multiplication and division we get rid of the nominator and of constant A. Thus a simple relation is remaining

$$-\frac{2}{a^3} + \frac{2a}{S^2} = 0$$

From this expression we have to find a:

$$\frac{2a}{S^2} = \frac{2}{a^3}$$

$$a^4 = S^2$$

$$a^2 = S$$

$$a = \sqrt{S}$$

and using the above substitution

$$b = \frac{S}{a} = \frac{S}{\sqrt{S}} = \frac{(\sqrt{S})^2}{\sqrt{S}} = \sqrt{S}$$

We find $a = b$. I.e., the highest frequency is gained for a quadratic sound plate with $a = b = \sqrt{S}$.

Solution of Ex. 5 continued:

$$f = A\sqrt{\frac{1}{a^2} + \frac{a^2}{s^2}}$$

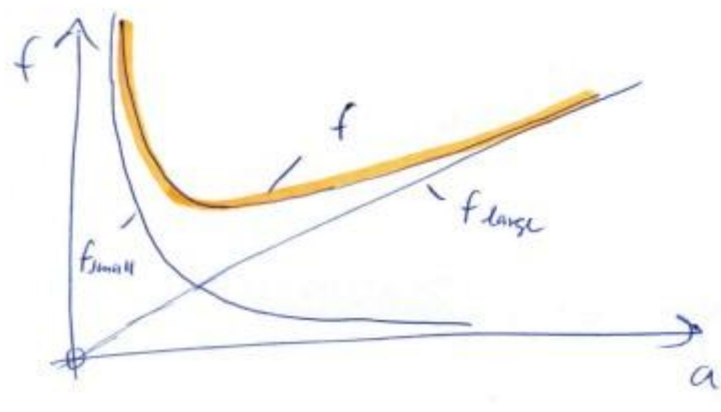
Approximation for small a : $\frac{a^2}{s^2} \ll \frac{1}{a^2}$

$$f_{small} = A\sqrt{\frac{1}{a^2}} = \frac{A}{a} \rightarrow \text{hyperbola}$$

Approximation for large a : $\frac{a^2}{s^2} \gg \frac{1}{a^2}$

$$f_{large} = A\sqrt{\frac{a^2}{s^2}} = A\frac{a}{s} \rightarrow \text{linear function}$$

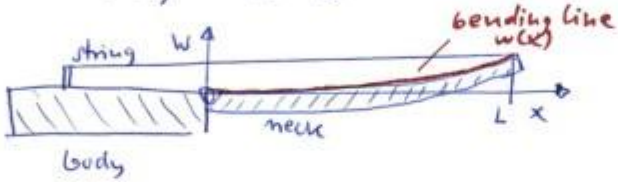
Sketch:



Solution of Ex. 6

1) Find the value of the second derivative of the bending line w

$$w'' = \frac{M}{E \cdot I_y} = \frac{6}{10^{10} \cdot 10^{-7}} = 6 \cdot 10^{-3} = 0.006 \qquad \left[\frac{Nm\, m^2}{N\, m^4}\right] = \left[\frac{1}{m}\right]$$



Note: There is an error in the units of I_y in the exercise. Should be m^4 instead m^3.

2) Integrate first time:

First derivative w' can be gained by integration of the second derivative, which is in this case a constant function.

$w'' = 0.006$

$w' = 0.006 \cdot x + C$ ... C is integration constant ...

We find C from the condition that gradient of the bending line at the position where the neck is clamped into the body should be zero.

$w'(0) = 0 = 0.006 \cdot 0 + C = 0 + C \rightarrow C = 0$

Now $w' = 0.006 \cdot x$ which is a simple linear function

3) Integrate second time:

The bending line $w(x)$ is found by another integration of the first derivative

$w' = 0.006 \cdot x$

$w = \int 0.006\, x\, dx = 0.006 \cdot \frac{x^2}{2} + D$ ... D is integration constant ....

D is found from condition of no bending at clamping point

$w(0) = 0.006 \cdot \frac{0^2}{2} + D = 0 + D = 0 \rightarrow D = 0$

$\rightarrow w(x) = 0.003\, x^2$

Bending at $L = 0.4\,m$ is $w(L) = 0.003 \cdot (0.4)^2 = 0.00048 \simeq \underline{\underline{0.5\ mm}}$

Solution of Ex. 7

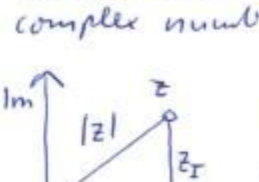
First of all: sorry, this is a little bit tough, but anyway not really difficult!

$$A = \frac{1}{|(R+i\omega L)\,i\omega C+1|} \quad \text{... multiply } i\omega C \text{ into the brackets}$$

$$= \frac{1}{|(i\omega RC - \omega^2 LC + 1)|} \quad \text{... order according to real and imaginary part}$$

$$= \frac{1}{|(1-\omega^2 LC) + i\omega RC|} \quad \text{... use rule for absolute value of complex number } z \text{ from Pythagoras}$$

$$= \frac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$



$z = z_R + i z_I$

$|z| = \sqrt{z_R^2 + z_I^2}$

Now we have found the function $A(\omega)$ for the amplification factor of the pick-up depending on the electrical properties L, R and C which are given. Angular frequency ω is related to acustic frequency f by $f = 2\pi\omega$. Thus we keep ω for further calculations and switch to f in the end.

To calculate the maximum of the function $A(\omega)$ where we have the resonance point with highest amplification we set first derivative $\frac{dA}{d\omega} = 0$ as usual for min-max-exercises.

To find the derivative we write $A(\omega)$ in more convenient form using that square root in nominator can be written as power $(-\frac{1}{2})$

$$A(\omega) = \left[(1-\omega^2 LC)^2 + \omega^2 R^2 C^2\right]^{-\frac{1}{2}}$$

Solution of Ex. 7 continued

For calculation we need chain rule $f'(x) = f'[u(x)] = \frac{df}{du} \cdot \frac{du}{dx}$.

In our case u corresponds to $u = (1-\omega^2 LC)^2 + \omega^2 R^2 C^2$, i.e. we have

$$A(\omega) = [u(\omega)]^{-\frac{1}{2}}$$

$$A'(\omega) = -\frac{1}{2}[u]^{-\frac{3}{2}} \cdot \frac{du}{d\omega}$$

$$A'(\omega) = -\frac{1}{2}[u]^{-\frac{3}{2}} \cdot \left[2(1-\omega^2 LC)\cdot(-2\omega LC) + 2\omega R^2 C^2\right]$$

$$= \frac{(4\omega LC(1-\omega^2 LC) - 2\omega R^2 C^2)}{2\cdot[u]^{\frac{3}{2}}} = 0$$

By multiplying with $2[u]^{\frac{3}{2}}$ on both sides the denominator cancels out and the equation becomes much simpler

$$4\omega LC(1-\omega^2 LC) - 2\omega R^2 C^2 = 0 \qquad | : 2\omega C$$

Division by $2\omega C$ further simplifies things

$$2L(1-\omega^2 LC) - R^2 C = 0$$

Now we can solve for ω by some rearrangements

$$2L(1-\omega^2 LC) = R^2 C$$

$$1 - \omega^2 LC = \frac{R^2 C}{2L}$$

$$\omega^2 LC = 1 - \frac{R^2 C}{2L}$$

$$\omega^2 = \frac{1 - R^2 C/2L}{LC}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f > \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

≈ 0 if R and C small

$$\rightarrow \omega = \sqrt{\frac{1}{LC}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

for single coil:

$$f \approx \frac{1}{2\pi}\sqrt{\frac{1}{3.8\cdot 130\cdot 10^{-12}}} = 7160\,\text{Hz}$$

for humbucker:

$$f \approx \frac{1}{2\pi}\sqrt{\frac{1}{12{,}6\cdot 100\cdot 10^{-12}}} = 4480\,\text{Hz}$$

Solution of Ex. 8

x : number of guitars (6 strings per instrument)
y : number of basses (4 strings per instruments)

$x + y = 9$ ... total number ...

$1 \cdot 6 \cdot x + 2 \cdot 4 \cdot y = 62$ ... total price ...

Thus we have two equations with two unknowns x and y.

$$x + y = 9 \qquad (1)$$
$$6x + 8y = 62 \qquad (2)$$

From (1) we get $y = 9 - x$, which we introduce into (2)

$$6x + 8(9 - x) = 62$$
$$6x + 72 - 8x = 62$$
$$72 - 62 = 8x - 6x$$
$$10 = 2x$$
$$x = 5 \quad \rightarrow \underline{\underline{5 \text{ guitars}}}$$

From (1) $y = 9 - 5 = 4 \quad \rightarrow \underline{\underline{4 \text{ bass guitars}}}$

Solution of Ex. 9

x : number of guitars (6 strings, 3 pickups)
y : number of 4-string basses } 2 pickups
z : number of 5-string basses

$x + y + z = 9$ (1) ... from total number

$6 \cdot x + 2 \cdot 4 \cdot y + 2 \cdot 5 \cdot z = 68$ (2) ... from total cost of strings

$3 \cdot 30x + 2 \cdot 40y + 2 \cdot 40z = 760$ (3) ... from total cost of pickups

Thus we have three equations for three unknowns

$x + y + z = 9 \rightarrow x = 9 - y - z$ (4)

$6x + 8y + 10z = 68$ (5)

$90x + 80y + 80z = 760$ (6)

Introduce (4) into (5) and (6)

$6(9 - y - z) + 8y + 10z = 54 - 6y - 6z + 8y + 10z = 68$ (7)

$90(9 - y - z) + 80y + 80z = 810 - 90y - 90z + 80y + 80z = 760$ (8)

Put together all the terms with y, z and absolute values

$2y + 4z = 14$ | : 2 ... to simplify further
$10y + 10z = 50$ | : 10

$y + 2z = 7$ (9)
$y + z = 5 \rightarrow y = 5 - z$ (10)

Introduce (10) into (9)

$5 - z + 2z = 5 + z = 7 \rightarrow z = 7 - 5 = 2$ 5-string basses

$y = 5 - z = 5 - 2 = 3$ 4-string basses from (10)

$x = 9 - y - z = 9 - 3 - 2 = 4$ guitars from (4)

Solution of Ex. 10

x : number of beer drinkers
y : number of wine drinkers
z : number of juice drinkers

$x + y + z = 11$ (1) ... from total number
$3x + y + 2z = 24$ (2) ... from total of liters
$0{,}5x + 0{,}5y = 4$ ... from total of cigarettes $\rightarrow 0{,}5\,y = 4 - 0{,}5\,x$ ... use simplest equation to get rid of variable y
$y = 8 - x$ (3)

Introduce (3) in (1) and (2)

$x + 8 - x + z = 11 \rightarrow 8 + z = 11 \rightarrow \underline{z = 3}$ (4) juice drinkers
$3x + 8 - x + 2z = 24 \rightarrow 2x + 2z = 16$ (5)

Introduce (4) into (5)

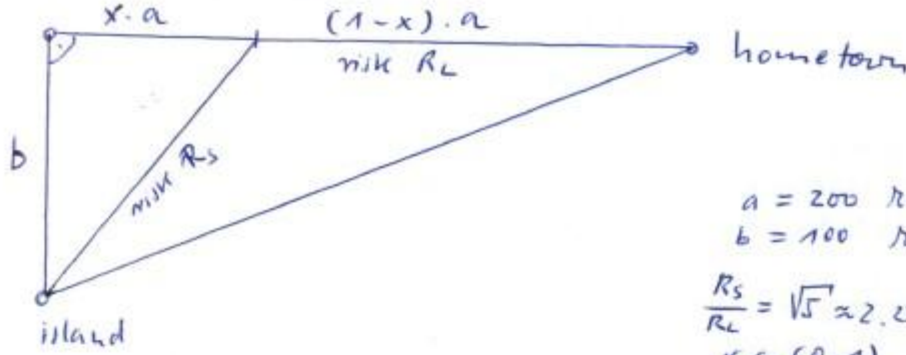
$2x + 2 \cdot 3 = 16$
$2x = 10 \rightarrow x = \underline{\underline{5}}$ beer drinkers

And finally from (3) $y = 8 - x = 8 - 5 = \underline{\underline{3}}$ wine drinkers

The result shows that beer is the most popular drink. This indicates that the exercise used very realistic conditions.

Solution of Ex. 11

Make a drawing for better imagination.



$a = 200$ km
$b = 100$ km
$\frac{R_S}{R_L} = \sqrt{5} \approx 2.236$
$x \in (0,1)$

The risk is given as risk per km. Thus the total risk is risk per kilometer times distance. Along the coast the travelling distance is $(1-x) \cdot a$. Over the sea we can calculate from Pythagoras law the distance as $\sqrt{x^2a^2+b^2}$. Thus we get for total risk

$$y(x) = R_L \cdot a(1-x) + R_S\sqrt{x^2a^2+b^2}$$

To find the minimum risk we must set first derivative to zero.

$$y' = -R_L \cdot a + R_S \frac{1 \cdot \cancel{2}xa^2}{\cancel{2}\sqrt{x^2a^2+b^2}} = 0 \qquad | : a \quad | \cdot \sqrt{x^2a^2+b^2}$$

$$-R_L\sqrt{x^2a^2+b^2} + R_S \, x \cdot a = 0$$

$$R_S \cdot x \cdot a = R_L\sqrt{x^2a^2+b^2} \qquad \text{... divide by } R_L \text{ ... } \text{.... square ...}$$

$$\left(\frac{R_S}{R_L}\right)^2 x^2a^2 = x^2a^2 + b^2$$

$$x^2a^2\left[\left(\frac{R_S}{R_L}\right)^2 - 1\right] = b^2$$

$$x^2 = \frac{b^2}{a^2\left[\left(\frac{R_S}{R_L}\right)^2-1\right]}$$

$$x = \frac{b}{a\sqrt{\left(\frac{R_S}{R_L}\right)^2-1}} = \frac{100}{200\sqrt{5-1}} = \frac{1}{4}$$

Result:
The musician should travel 150 km along the coast and then embark for minimum risk

Solution of Ex. 12

a) $1+3+5+7+\dots+(2n-1)=n^2$

This holds for $n=1$, since $1=1$. We assume that it holds for n as written above and show that based on this it also holds for $n+1$

$$\underbrace{1+3+\dots+(2n-1)}_{=n^2 \text{ since we assume that relation holds for } n}+(2n+1)=(n+1)^2$$

$$n^2+2n+1=(n+1)(n+1)=(n+1)^2 \quad \checkmark$$

This is obviously an identity and thus a true statement. This means that relation also holds for $n+1$. Since we can proceed in this way for each "next n" starting from true relation for $n=1$, this holds always!

b) $1^2+2^2+3^2+\dots+n^2=n(n+1)(2n+1)/6$

This holds for $n=1$, since $1=1\cdot2\cdot3/6=1$.
We assume it holds for arbitrary n, write the expression for next n, which is $n+1$ and show that we get a true expression using the relation for n.

$$\underbrace{1^2+2^2+3^2+\dots n^2}_{=n(n+1)(2n+1)/6}+(n+1)^2=(n+1)\underbrace{(n+2)(2n+3)}/6$$

$$\frac{n(n+1)(2n+1)}{6}+(n+1)^2= \qquad \frac{(n+1)}{6}\underbrace{(2n^2+4n+3n+6)}$$

$$\frac{(n+1)}{6}\left[\underbrace{n(2n+1)+6(n+1)}\right]= \qquad \left(\frac{n+1}{6}\right)(2n^2+7n+6)$$

$$\frac{n+1}{6}\cdot\underbrace{(2n^2+n+6n+6)}=$$

$$\frac{n+1}{6}\cdot(2n^2+7n+6)$$

We have on both sides the same expressions. Thus the relation is also true for $n+1$ and therefore always!

Solution of Ex. 12 continued / 1

c) $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

This holds for $n=1$, since $1 = \left(\frac{1.2}{2}\right)^2 = 1$

Now we have to show that the expression for $n+1$ is true assuming it holds already for n.

$$\underbrace{1^3+2^3+3^3+\dots+n^3}_{\left[\frac{n(n+1)}{2}\right]^2}+(n+1)^3 = \left[\frac{(n+1)(n+2)}{2}\right]^2$$

$$\frac{n^2(n+1)^2}{(2)^2} + (n+1)^3$$

$$= (n+1)^2\left(\frac{n^2}{4} + n+1\right)$$

$$= (n+1)^2 \frac{n^2+4n+4}{4}$$

$$= (n+1)^2 \frac{(n+2)^2}{2^2}$$

$$= \left[\frac{(n+1)(n+2)}{2}\right]^2$$

... same as right hand side, thus always true

d) $5^n + 2^{n+1} = 3m$ with $m \in \mathbb{N}$

$n=1$: $5+2^2 = 5+4 = 9 = 3 \cdot 3$ is obviously true.

Now we write the expression for $n+1$ and modify it somehow to retrieve the expression for n, which we assume to be valid. Here we use the trick of adding and subtracting a certain expression, which helps to isolate the already valid „$5^n + 2^{n+1}$".

$$5^{n+1} + 2^{n+2} = 5^n 5 \underbrace{+2^{n+1}.2 + \overbrace{2^{n+1}.3 - 2^{n+1}.3}^{0}}$$

$$= 5^n.5 + 2^{n+1}.5 \qquad -2^{n+1}.3$$

$$= 5(5^n+2^{n+1}) - 2^{n+1}.3$$

$$= 5 \cdot 3m - 2^{n+1}.3 = 3(5m - 2^{n+1})$$ is aswell multiple of 3 ✓

Solution of Ex. 12 continued /2

e) The statement that $2^{(2^n)}+1$ has always 7 as last digit can be written as

$2^{(2^n)} + 1 = 10 \cdot m + 7$ with $m \in \mathbb{N}$ being a natural number

If we test for $n = 2$ we get

$2^{2^2} + 1 = 2^4 + 1 = 16 + 1 = 17$, which is a number with 7 as last digit.

Now we assume that the relation holds for arbitrary n and show that based on this it also holds for n+1.

$$2^{2^{n+1}} + 1 = 2^{2^n \cdot 2} + 1$$

$$= \left(2^{2^n}\right)^2 + 1$$

... now we use the relation for n $2^{2^n}+1 = 7 + 10m$ which can be written as $2^{2^n} = 6 + 10m$ as well.

$$= (6 + 10m)^2 + 1$$

$$= 36 + 120m + 100m^2 + 1$$

$$= 37 + 120 \cdot m + 100m^2$$

$$= 30 + 12 \cdot 10 \cdot m + 10 \cdot 10 \cdot m^2 + 7$$

$$= 10 \cdot (3 + 12m + 10m^2) + 7 = 10 \cdot \tilde{m} + 7$$

This is also a natural number with 7 as last digit!

Solution of Ex. 13

From physics we know that velocity is a vector, which can be decomposed into components in x and y direction according to the coordinate system defined in the sketch drawn for this exercise. Thus v_0 can be split into v_{0x} and v_{0y}. In x-direction the guitar moves with constant velocity, in y-direction there is a superposition of a uniform upward movement with velocity v_{0y} plus an accelerated downward movement according to gravity. As explained in the sketch this yields the parabolic movement of the flying guitar.

$v_{0x} = v_0 \cdot \cos(53.13°) = 15 \frac{m}{s}$

$v_{0y} = v_0 \cdot \sin(53{,}13°) = 20 \frac{m}{s}$; $g = 10 \frac{m}{s^2}$

$$y = \frac{4}{3}x - \frac{2}{90}x^2 = x \cdot \overbrace{\left(\frac{4}{3} - \frac{2}{90}x\right)}^{=0} = 0$$

From the above equation the zero-values for y (i.e. start and end point of flight) are easily found

$x_1 = 0\,m$; $x_2 = \frac{4 \cdot 90}{3 \cdot 2} = 60\,m$

Thus the distance to the guy catching the guitar in the auditory is 60 m.

For symmetry reasons the horizontal distance between guitar player and highest point is 30 m.

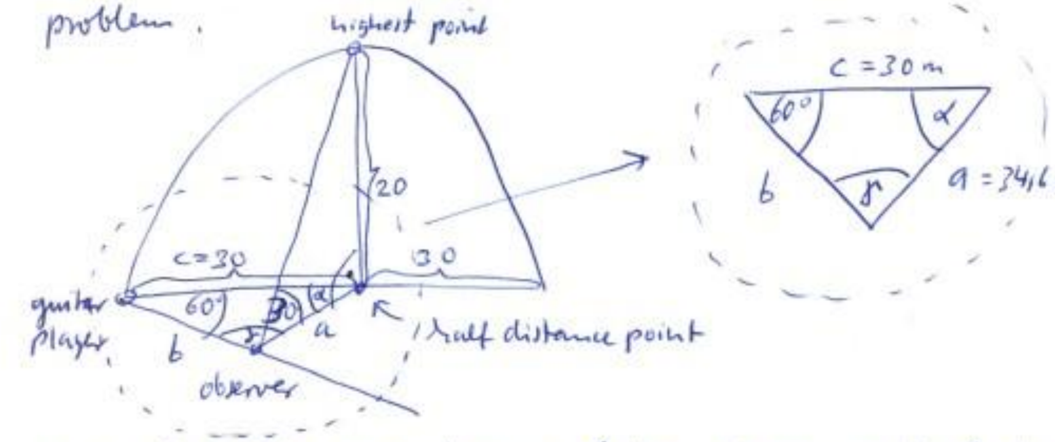
To find the maximum height of the flying guitar we have to set first derivative of the parabola to 0,

$y' = \frac{4}{3} - \frac{4}{90}x = 0 \rightarrow x = 30$ (see above) and

$y(30) = \frac{4}{3} \cdot 30 - \frac{2 \cdot 30^2}{90} = 20\,m$

Solution of EX. 13, continued

Now we have found some more geometrical information about distances between characteristic points of our problem.



From the rectangular triangle between observer and highest point and „half distance point" we can calculate the distance a

$$\frac{a}{20\,m} = \cot 30^\circ \rightarrow a = 20\,m \cdot \cot 30^\circ = 34.6\ m$$

From „law of sines" we can find angle γ which we need to determine angle γ, which furthermore is needed to find distance b from „law of cosines"

$$\frac{\sin \gamma}{c} = \frac{\sin 60^\circ}{a} \rightarrow \sin \gamma = \frac{c}{a} \sin 60^\circ$$

$$= \frac{30}{34.64} \cdot \sin 60^\circ = 0.75$$

$$\gamma = \arcsin 0.75 = 48{,}59^\circ$$

From sum of angles in triangle to be 180° we can find α.

$$\alpha = 180^\circ - 60^\circ - \gamma = 71{,}41^\circ$$

Now we know α and can calculate distance b between guitar player and observer from „law of cosines"

$$b^2 = a^2 + c^2 - 2ac\cos\alpha$$

$$b = \sqrt{34.6^2 + 30^2 - 2 \cdot 30 \cdot 34.6 \cdot \cos 71.41^\circ} = \underline{\underline{42{,}06\ m}}$$